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复杂区域内粘性流动的数值计算

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本文应用原始变量法 (SIMPLEC) 计算了复杂区域内的不可压和可压缩流动问题。计算时, 仍使用交错网格, 但在求解动量方程时采用协变分量做为原始变量, 有效地消除了在任意非正交坐标系下压力和笛卡尔速度分量不耦合而产生的压力波动。在压力修正方程中考虑了密度的影响并构造出一种密度的迎风格式, 保证了计算的稳定与收敛, 使之预示马赫数变化范围较大的流动成为可能。对由 NACA65- ($12A_{10}$) $_{10}$构成的平面叶栅、由 NGTE$10C_4/30C_{50}$ 构成的平面叶栅的不可压缩流动和平面超声速缩放喷管进行了计算。结果表明, 本方法有较好的收敛性和准确性, 能够适用于从亚音速到超音速的流动场合, 有一定实用价值。

一、采用协变分量的离散方程

在任意非正交曲线坐标 ξ-η 下的通用方程为:

$$\frac{1}{J}\frac{\partial}{\partial\xi}(\rho\bar{U}\Phi)+\frac{1}{J}\frac{\partial}{\partial\eta}(\rho\bar{V}\Phi)=\frac{1}{J}\frac{\partial}{\partial\xi}\left[\frac{\Gamma_\Phi}{J}(\alpha\Phi_\xi-\beta\Phi_\eta)\right]+\frac{1}{J}\frac{\partial}{\partial\eta}\left[\frac{\Gamma_\Phi}{J}(\gamma\Phi_\eta-\beta\Phi_\xi)\right]+S_\Phi \quad (1)$$

其中　$\bar{U}=uy_\eta-vx_\eta$, $\bar{V}=vx_\xi-uy_\xi$, $\alpha=x_\eta^2+y_\eta^2$, $\beta=x_\xi x_\eta+y_\xi y_\eta$, $\gamma=x_\xi^2+y_\xi^2$, $J=x_\xi y_\eta+y_\xi x_\eta$, Φ是笛卡坐标系下的任一输运量, S_Φ 是输运量 Φ 在 ξ-η 坐标下的源项。

向量 $\vec{V}(x, y)$ 在等 ξ、η 线上的投影分量 $V_{\bar{t}(\eta)}$ 和 $V_{\bar{t}(\xi)}$ 分别与 η 线和 ξ 线的方向一致 (图 1)。可以得到:

$$V_{\bar{t}(\eta)}=(x_\xi u+y_\xi v)/\sqrt{\gamma}=u_\xi/\sqrt{\gamma}$$
$$V_{\bar{t}(\xi)}=(x_\eta u+y_\eta v)/\sqrt{\alpha}=v_\eta/\sqrt{\alpha} \quad (2)$$

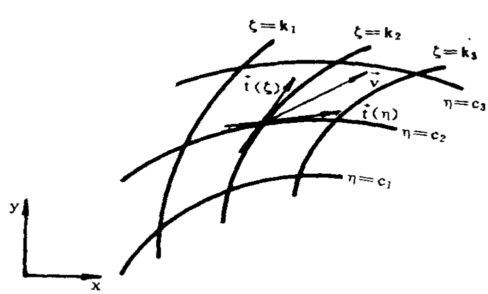

图 1　任意非正交坐标系

在张量分析中, $V_{\bar{t}(\eta)}$ 和 $V_{\bar{t}(\xi)}$ 是 $\vec{V}$ 的协变物理分量, 而 u_ξ 和 v_η 是 $\vec{V}$ 的协变分量。本文采用 u_ξ、v_η 做为原始变量得到如下离散方程。

动量方程

$$a_p u_{\xi,p}=\sum a_{nb}u_{\xi,nb}+A_\xi(P_\omega-P_e)+b_{u,\xi}, \quad a_p v_{\eta,p}=\sum a_{nb}v_{\eta,nb}+A_\eta(P_s-P_n)+b_{v,\eta} \quad (3)$$

其中

$$A_\xi=(J\Delta\eta)_p \quad A_\eta=(J\Delta\xi)_p$$

$$b_{u,\xi}=b_u(x_\xi)_p+b_v(y_\xi)_p+a_E(u'_{\xi,E}-u_{\xi,E})+a_W(u'_{\xi W}-u_{\xi W})+a_N(u'_{\xi N}-u_{\xi,N})+a_S(u'_{\xi S}-u_{\xi S})$$

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$$b_{v,\eta} = b_u(x_\eta)_p + b_v(y_\eta)_p + a_E(v'_{\eta,E} - v_{\eta,E}) + a_W(v'_{\eta,W} - v_{\eta,W}) + a_N(v'_{\eta,N} - v_{\eta,N}) + a_S(v'_{\eta,S} - v_{\eta,S})$$

而 $u'_\xi = u'(x_\xi)_p + v'(y_\xi)_p$, $v'_\eta = u'(x_\eta)_p + v'(y_\eta)_p$, $u_\xi = ux_\xi + vy_\xi$，$v_\eta = ux_\eta + vy_\eta$

压力修正方程[1] $A_P P'_P = A_E P'_E + A_W P'_W + A_N P'_N + A_S P'_S + b_1 + b_2$　　(4)

在可压缩流动的情况下，马赫数取代了 Peclet 数而成为流动的控制参数，这时应使压力修正方程在亚音速情况下显示出椭圆性质，在超音速情况下显示出双曲型性质。为此应对密度使用一阶迎风格式来求界面的值，对压力则仍使用中心差分，故控制体界面的密度和其修正值表示如下:

$$\rho_e = \rho_P \max[u_{\xi e}, 0]/u_{\xi e} + \rho_E \max[-u_{\xi e}, 0]/u_{\xi e} \qquad (5a)$$

$$\rho'_e = KP'_P \max[u_{\xi e}, 0]/u_{\xi e} + KP'_E \max[-u_{\xi e}, 0]/u_{\xi e} \qquad (5b)$$

采用 (5a) 和 (5b) 式之后，方程 (4) 的系数变成:

$$A_E = (c_1\Delta\eta)_e(\rho^*_e d u_{\xi,e} + K_E \max[-u^*_{\xi,e}, 0]), \quad A_W = (c_1\Delta\eta)_w(\rho^*_w d u_{\xi,\omega} + K_W \max[u^*_{\xi,w}, 0])$$

$$A_N = (c_2\Delta\xi)_n(\rho^*_n d u_{\eta,n} + K_N \max[-v^*_{\eta,n}, 0]), \quad A_S = (c_2\Delta\xi)_s(\rho^*_s d u_{\eta,s} + K_S \max[v^*_{\eta,s}, 0])$$

$$A_P = (c_1\Delta\eta)_e(\rho^*_e d u_{\xi,e} + K_P \max[u^*_{\xi,e}, 0]) + (c_1\Delta\eta)_w(\rho^*_w d u_{\xi,\omega} + K_P \max[-u^*_{\xi,w}, 0]) + (c_2\Delta\xi)_n(\rho^*_n d u_{\eta,n} + K_P \max[v^*_{\eta,n}, 0]) + (c_2\Delta\xi)_s(\rho^*_s d u_{\eta,s} + K_P \max[-v^*_{\eta,s}, 0])$$

$$b_1 + b_2 = (\rho^*\Delta\eta\bar{U})_w - (\rho^*\Delta\eta\bar{U})_e + (\rho^*\Delta\xi\bar{V})_s - (\rho^*\Delta\xi\bar{V})_n \qquad (6)$$

其中，$c_1 = \alpha/J$，$c_2 = \gamma/J$，$c_{12} = \beta/J$，$\bar{U} = c_1 u_\xi - c_{12} v_\eta$，$\bar{V} = c_2 v_\eta - c_{12} u_\xi$，$du_\xi = A_\xi/(a^u_p - \Sigma a^u_{nb})$，

$dv_\eta = A_\eta/(a^v_p - \Sigma a^v_{nb})$。

$K = \partial\rho/\partial P$，它反映了压力对密度的影响，其值由状方程求得为 $K = 1/RT$。对于不可压缩流动，由 $K = \partial\rho/\partial p$ 得 $K = 0$，方程 (4) 就退化为不可压缩流动的压力修正方程。

离散方程组的求解　给出的封闭方程加上定解条件，即可求解可压和不可压流动问题。当计算平面叶栅内不可压湍流时用标准 k-ε 双方程模型求 μ_t 并在壁面附近使用壁面律[1,2]。在计算可压流时，假定滞止焓等于常数，以此替代能量方程。

二、计算结果与讨论

图 2 和图 3 分别是 NACA65-(12A$_{10}$)$_{10}$叶栅和 NGTE10C$_4$/30C$_{50}$叶栅的计算值和实验值的比

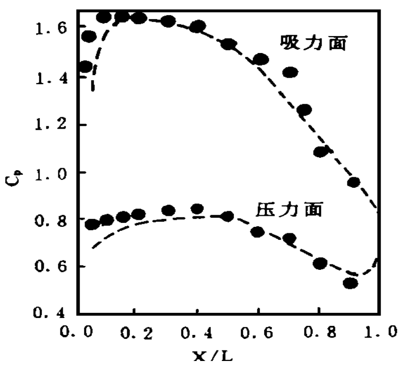

图 2　叶栅的压力系数 c_p

(-- 本文计算，• 实验[3]，Re= 2.8× 10^5)

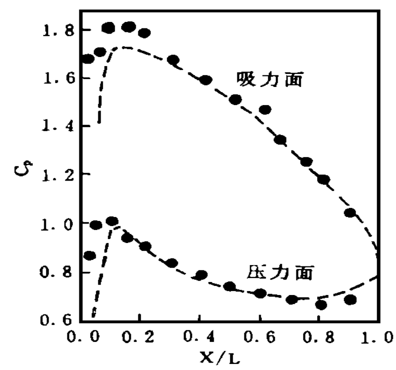

图 3　叶栅的压力系数 c_p

(-- 本文计算，• 实验[3]，Re= 4.0× 10^5)

较，进口气流角 β_1 均为 30°。网格由 45× 21 的 H 形网格构成，迭代 150 次就得到收敛解。因来流的马赫数较低，计算是按不可压情况计算的。可以看出：计算值和实验值的吻合程度是令人满意的，但两种叶栅的前缘处误差稍大些，且由于 NGTE 的前缘半径比 NACA 的大，因而误差也大。图 4 是文献 [4] 提供的平面缩放喷管 (*B*-1)，因对称性只取一半做为计算域，网格由代数法生成为 21× 11，喉部与壁面附近的网格点要密些。入口马赫数为 0.232，背压与入口滞止压力 p^* 之比为 0.1135，*Re* 取为 7.5×10^6。由图 5、图 6 看出计算结果与实验结果吻合得相当好。在喉部下游曲率半径不连续处，超音速气流过度膨胀后，又产生压缩波引起的壁面附近压力回升，气流速度在喉部壁面附近的升高要比中心线附近来得快，从而导致等马赫线的弯曲 (见图 7)。

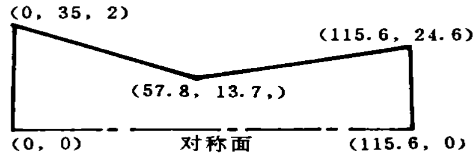

图 4　缩放喷管的几何尺寸 (mm)

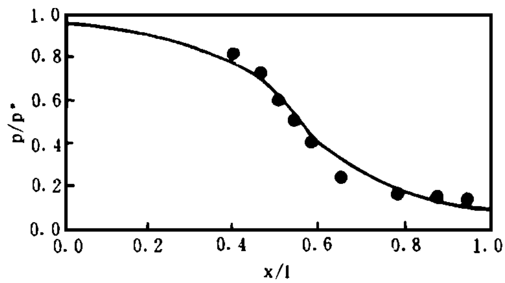

图 5　沿中心线的压力分布

(—本文计算，• 文献 [4] 实验)

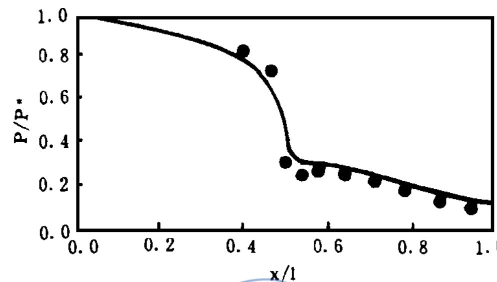

图 6　沿壁面的压力分布

(—本文计算，• 文献 [4] 实验)

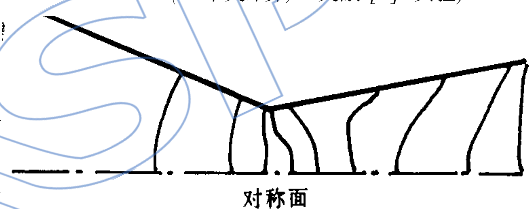

图 7　等马赫线的分布 (*Ma* 从 0.4~ 2.0)

结论　(1) 用 SIMPLEC 方法直接求 N-S 方程将亚音速与超音速同时存在的流动统一起来进行计算是可行的，程序具通用性和可靠性。(2) 在动量方程中，以速度的协变分量做为原始变量后，有效地消除了压力场的波动，具有良好的计算稳定性和收敛性。

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RESEARCH ON DISCHARGE COEFFICIENT OF COOLING HOLES ON FEEDER TUBE

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ABSTRACT The main air flow supplied in the feeder tube is delivered to cooling holes with axis perpendicular to the main flow. This crossflow pattern exerts a significant influence on the hole discharge coefficient. The experimental investigation of the hole discharge coefficients and the measurement of static pressure distribution along the tube axis have been completed. It is shown that the discharge coefficient increases as the ratio of the orifice thickness to diameter and the velocity head ratio between orifice flow and tube flow increase. But the discharge coefficient approaches to a constant when the velocity head ratio is greater than 100. Empirical formulas have been dedueed from the experimental data. The static pressure distribution along the tube and the air flow rate allocation for the holes have been calculated by using the general air system computer program developed by author. The results ageree with the experimental data well.

COORDINATION OF α、β COEFFICIENTS WITH *k* COMPLEXES FOR VARYING SPECIFIC HEAT SOLUTIONS

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ABSTRACT Three complexes ε_T, ε, ε_v of k are introduced to simplify the original expressions of gasdynamic functions with k of varying specific heats and their analytical solutions, and to unite two other forms of gasdynamic functions using coefficients α of β so that the calculations and applications at higher temperatures could be more convenient. It is clarified that the effect of varying specific heats reflected through k is the essence of these functions. The different sets of formulae are also briefly compared and analysed for reference in engineering applications.

NUMERICAL STUDY OF VISCOUS FLOWS IN COMPLEX GEOMETRIES

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ABSTRACT The program SIMPLEC has been adopted to calculate the viscous flows at all Mach numbers inside a plane cascad and nozzle. In order to tread both incompressible and compressible flows, pressure is chosen as a primary dependent variable in preference to density. The effect of the pressure on the density is implicitly included in the pressure correction equation. Further, the covariant velocity components are chosen as the dependent variables in the momentum equations, which makes the pressure-velocity coupling relatively easy to handle. Comparison of the calculated results with the experimental data show that the present method exibits much better stability and covergence behavior, and is applicable to a variety of the incompressible and compressible flows, ranging from subsonic to supersonic.